SETON SCHOOL
SUMMER MATH REFRESHER PROGRAM

GEOMETRY REVIEW

Many studies have shown that, during the summer, students lose a substantial portion of their math skills acquired over the course of a school year. This puts them at a disadvantage upon returning for a new school year, as the expectations of a new course presuppose the skills and knowledge taught in the previous course.

The Seton math department, in cooperation with the administration, has a program similar to the summer reading program for English. Our hope is that this program will help students transition out of the summer recess and into their new math courses smoothly and with less stress.

We recommend that the sessions be worked gradually over several weeks, perhaps one session done in a day, and two or three days a week. There are ten sessions, each one of which should take about a half hour, though the times will undoubtedly vary among students.

As with summer reading, the work is expected to be done before the new school year starts. You are responsible for keeping your papers and having them ready to turn in the first day. The summer work will count for approximately 5% of your first quarter grade in the next math course. (For this review, that is usually Algebra II.)

IMPORTANT: It is essential that you show all your work, and that it is organized and legible. Space has been provided for you to work directly on the packet, but you may attach extra loose leaf pages if necessary. You must fill in the answer boxes for each question. Put your name clearly on each page of work. If these conditions are not met, you will not get full credit for your work. Also, I strongly suggest that you scan or copy your papers, so that if you lose your originals, you will have a backup.

GEOMETRY REVIEW HELP PAGES

Solids (Surface Area and Volume)

Prism:	$S = 2B + Ph,$	$V = Bh$
Cylinder:	$S = 2\pi r^2 + 2\pi rh,$	$V = 2\pi r^2 h$
Pyramid:	$S = B + \frac{1}{2}PL,$	$V = \frac{1}{3}Bh$
Cone:	$S = \pi r^2 + \pi rL,$	$V = \frac{1}{3}\pi r^2 h$
Sphere:	$S = 4\pi r^2,$	$V = \frac{4}{3}\pi r^3$

Area

Circle:	$A = \pi r^2$
Trapezoid:	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram:	$A = bh,$ where $h \perp b$
Regular Polygon:	$A = \frac{1}{2}aP$

A = area, S = surface area, V = volume,
P = perimeter, h = height, B = area of base,
L = slant height, a = apothem

Properties of Exponents

Given $a \neq 0$ and $m, n > 0$:

Product Rule:	$a^m \cdot a^n = a^{m+n}$
Quotient Rule:	$\frac{a^m}{a^n} = a^{m-n}$
Zero Exponent Rule:	$a^0 = 1$
Negative Exponent Rule:	$a^{-m} = \frac{1}{a^m}$
Power of a Power Rule:	$(a^m)^n = a^{mn}$
Power of a Product Rule:	$(ab)^m = a^m b^m$
Power of a Quotient Rule:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Factoring Examples

Greatest Common Factor: $3x^2 - 12x = 3x(x-4)$

Difference of Perfect Squares: $9x^2 - 25 = (3x+5)(3x-5)$

Trinomial, $a = 1$: $x^2 - 3x - 4 = (x-4)(x+1)$

Trinomial, $a \neq 1$: $2x^2 + x - 6 = (2x-3)(x+2)$

Quadratic Formula

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Linear Equations

Slope-Intercept Form:	$y = mx + b$; where $m = slope$; $b = y\text{-}intercept$
Point-Slope Form:	$y - y_1 = m(x - x_1)$; where $m = slope$; (x_1, y_1) *a point on the line*
Standard Form:	$Ax + By = C$, where A, B, and C are integers
Vertical Line:	$x = a$ *constant*
Horizontal Line:	$y = a$ *constant*

Important Definitions

If three or more lines intersect at the same point, they are concurrent lines. The point of intersection is called the point of concurrency.

If *x* is the geometric mean of *a* and *b*, then $x^2 = ab$.

The median of a triangle is a segment with one endpoint at the midpoint of a side and the other endpoint at the vertex opposite the side.

A circle is circumscribed about a polygon if it contains all the vertices of the polygon. Note: Not all polygons with four or more sides can have a circumscribed circle.

A circle is inscribed in a polygon if it is tangent to each side of the polygon. Note: Not all polygons with four or more sides can have an inscribed circle.

Important Postulates and Theorems from Geometry

1. ***Vertical Angles Theorem (VAT)*** – Vertical angles are congruent.
2. ***Linear Pair Postulate (LPP)*** – If two angles form a linear pair, they are supplementary.
3. ***Corresponding Angles Postulate (CAP)*** – If two lines cut by a transversal are parallel, then corresponding angles are congruent. [The converse is also true.]
4. ***Alternate Interior Angles Theorem (AIAT)*** – If two parallel lines are cut by a transversal, then alternate interior angles are congruent. [The converse is also true.]
5. ***Alternate Exterior Angles Theorem (AEAT)*** – If two parallel lines are cut by a transversal, then alternate exterior angles are congruent. [The converse is also true.]
6. ***Consecutive Interior Angles Theorem (CIAT)*** – If two parallel lines are cut by a transversal, then consecutive interior angles are supplementary. [The converse is also true.]

Theorems and Postulates Concerning Triangles

7. ***Triangle Sum Theorem*** – The sum of the measures of the interior angles of a triangle is 180°.
8. ***Side-Side-Side Postulate (SSS)*** – If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
9. ***Side-Angle-Side Postulate (SAS)*** – If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
10. ***Angle-Side-Angle Postulate (ASA)*** – If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
11. ***Angle-Angle-Side Theorem (AAS)*** – If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding side of another triangle, then the triangles are congruent.
12. ***Triangle Inequality Theorem*** – The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
13. ***Exterior Angle Theorem*** – The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles of the triangle.
14. ***Base Angles Theorem*** – If two sides of a triangle are congruent, then the angles opposite them are congruent. [The converse is also true.]
15. ***Perpendicular Bisector Theorem*** – If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. [The converse is also true.]
16. ***Angle Bisector Theorem*** – If a point is on the bisector of an angle, then it is equidistant from the sides of the angle. [The converse is also true.]
17. ***Angle-Angle Similarity Postulate (AA)*** – If two angles of one triangle are congruent to two angles of another, then the two triangles are similar.
18. ***Side-Side-Side Similarity Theorem (SSS)*** – If the length of corresponding sides of two triangles are proportional, then the triangles are similar.
19. ***Side-Angle-Side Similarity Theorem (SSS)*** – If an angle of one triangle is congruent to an angle of another triangle, and the lengths of sides including these angles are proportional, then the triangles are similar.
20. ***Pythagorean Theorem*** – In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs. [The converse is also true.]

21. ***45°-45°-90° Triangle Theorem*** – In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

22. ***30°-60°-90° Triangle Theorem*** – In a 30°-60°-90° triangle, the hypotenuse if twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

23. If three parallel lines intersect two transversals, then they divide the transversal proportionally.

24. If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to each other and to the original triangle.

25. If an altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean of the two segments of the hypotenuse.

26. If an altitude is drawn to the hypotenuse of a right triangle, each leg of the original triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.

27. ***Concurrency of Perpendicular Bisectors of a Triangle*** – The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle. This point is called the <u>circumcenter</u>.

28. ***Concurrency of Angle Bisectors of a Triangle*** – The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle. This point is called the <u>incenter</u>.

29. ***Concurrency of Medians of a Triangle*** – The medians of a triangle intersect at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side. This point is called the <u>centroid</u>.

30. ***Concurrency of Altitudes of a Triangle*** – The lines containing the altitudes of a triangle are concurrent. The point of concurrency is called the <u>orthocenter</u>.

31. ***Midsegment Theorem*** – The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

32. If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side. [The converse is also true.]

Theorems Concerning Quadrilaterals

33. If a quadrilateral is a parallelogram, then its opposite sides are congruent. [The converse is also true.]

34. If a quadrilateral is a parallelogram, then its opposite angles are congruent. [The converse is also true.]

35. If a quadrilateral is a parallelogram, then its diagonals bisect each other. [The converse is also true.]

36. If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. [The converse is also true.]

37. If one pair of opposite sides of a quadrilateral are both congruent and parallel, then it is a parallelogram.

38. A parallelogram is a rhombus if and only if its diagonals are perpendicular.

39. A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

40. A parallelogram is a rectangle if and only if its diagonals are congruent.

41. A trapezoid is isosceles if and only if its diagonals are congruent.

42. ***Midsegment Theorem for Trapezoids*** – The midsegment of a trapezoid is parallel to both bases and its length is one half the sum of the lengths of the two bases.

43. If a quadrilateral is a kite, then its diagonals are perpendicular.

44. If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

Theorems Concerning Circles

45. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

46. In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

47. If two segments from the same exterior point are tangent to a circle, then they are congruent.

48. In the same or congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

49. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

50. If an angle is inscribed in a circle, then its measure is one-half the measure of its intercepted arc.

51. If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

52. A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

53. If a tangent and a chord intersect at a point on the circle, the measure of each angle formed is one-half the measure of its intercepted arc.

54. If two chords intersect inside a circle, the measure of each angle formed is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

55. If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

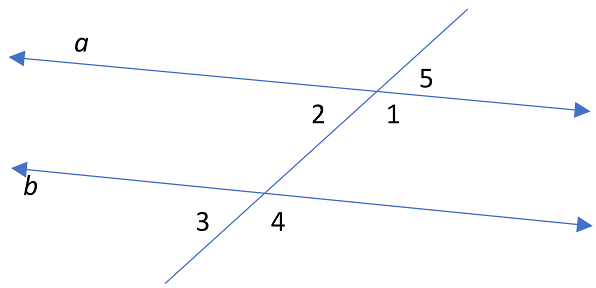
56. If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord.

57. If two secant segments share the same endpoint outside a circle, the product of the lengths of one secant segment and its external secant segment equals the product of the lengths of the other secant segment and its external secant segment.

58. If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

GEOMETRY REVIEW SESSION 1

1. If *a* is parallel to *b*, which <u>labeled</u> angle must be congruent to $\angle 1$?

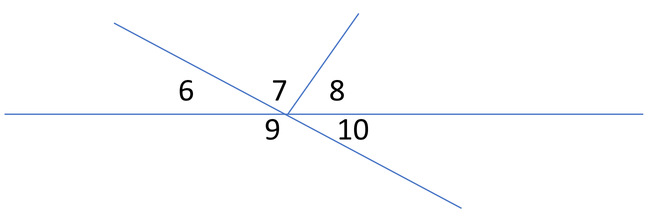


2. Complete the following theorem:
If two parallel lines are cut by a transversal, then pairs of consecutive interior angles are ________.

3. Write the converse of the following statement:
If it is raining, then people are not picnicking.

4. Given $m\angle 3 = 23°$, and given that $\angle 4$ and $\angle 5$ are, respectively, complementary and supplementary to $\angle 3$, what are the measures of $\angle 4$ and $\angle 5$?

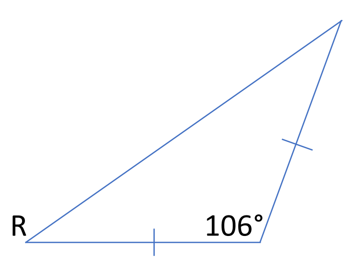
Use the figure below for questions 5 and 6. Answer each with a pair of the numbered angles.



5. Name a pair of vertical angles.

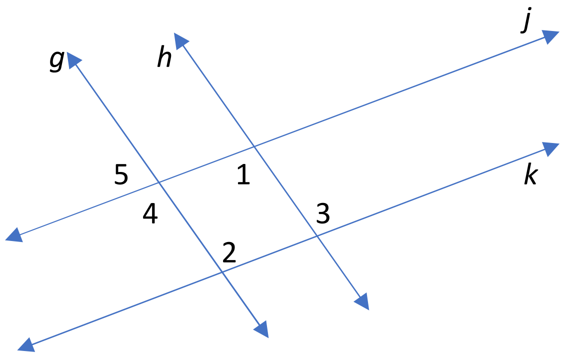
6. Name a linear pair.

7. Find $m\angle R$.



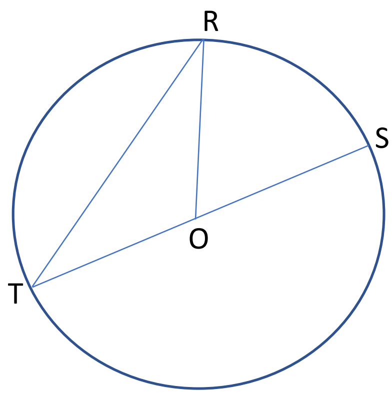
GEOMETRY REVIEW SESSION 1 (CONTINUED)

8. Given that $\angle 1 \cong \angle 3$, which two lines must be parallel? Why?



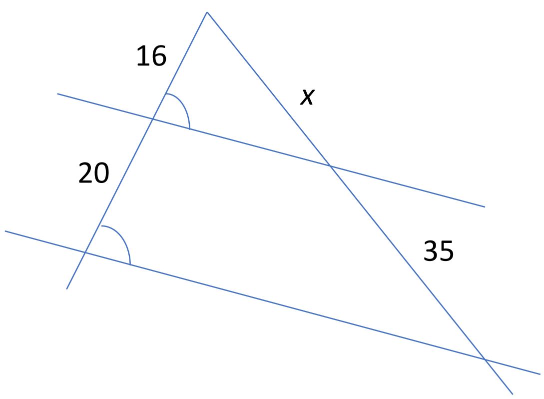
8.

9. In $\odot O$, $m\widehat{STR} = 290°$. *Find* $m\angle T$.



9.

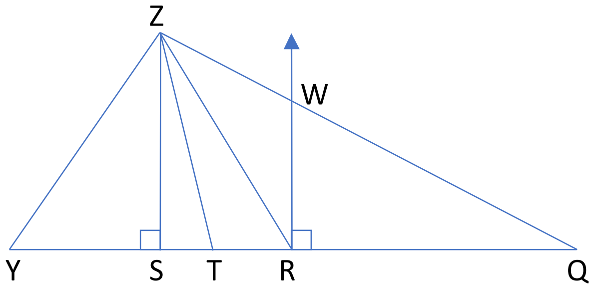
10. Find *x* in the figure below.



10.

GEOMETRY REVIEW SESSION 2

Use the figure below for questions 1-4.
Given: $YR = RQ,\ \angle YZT \cong \angle QZT$



1. Name an angle bisector of triangle QYZ.

2. Name a perpendicular bisector of $\overline{YQ}$.

3. Name a median of triangle QYZ.

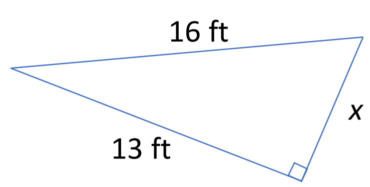
4. Name an altitude of triangle QYZ.

5. Can a triangle have sides of 41 m, 18 m, and 21 m? Explain.

6. Complete the theorem: *In a plane, if two lines are both perpendicular to a third line, then the two lines are _____________.*

7. The measure of an angle is 40 more than its supplement. What is the measure of each angle?

8. Find *x* in the following figure. You may leave answer in radical form.



9. Given a sphere with radius 5 cm, find the surface area <u>and</u> the volume. Leave both answers in terms of π.

10. Find the equation of a line which passes through $(7,2)$ and is parallel to $y = 2x + 4$.

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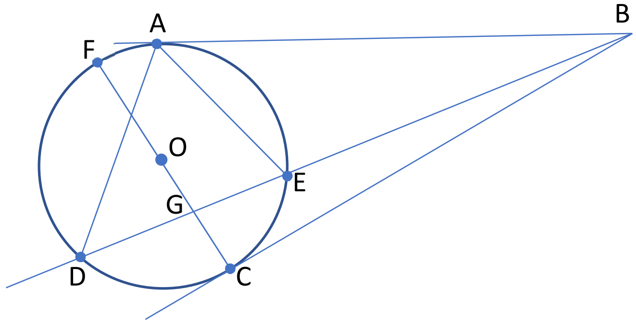
GEOMETRY REVIEW SESSION 3

Use ⊙ O and the figure below for all questions in this session 1-10.



1. Name each of the following from the figure.
   a. a secant **b**. a chord **c**. a tangent

2. If $m\angle ABC = 40°$, find $m\overset{\frown}{AEC}$.

3. Find $m\angle OCB$.

4. If $BD = 32$ and $BE = 18$, find AB.

5. If $BC = 17$, find AB.

6. If $m\overset{\frown}{AD} = 150°$ and $m\overset{\frown}{AE} = 80°$, find $m\angle ABD$.

7. If $m\overset{\frown}{DCE} = 110°$, find $m\angle DAE$.

8. If $EG = 10$, $DG = 12$, and $CG = 8$, find FG.

9. If $m\overset{\frown}{CE} = 45°$ and $m\overset{\frown}{FD} = 105°$, find $m\angle FGD$.

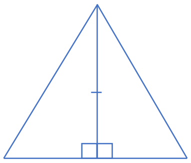
10. Name each of the following from the figure.
    a. diameter **b**. semicircle **c**. radius

1.a. 1.b. 1.c.
2.
3.
4.
5.
6.
7.
8.
9.
10.a. 10.b. 10.c.

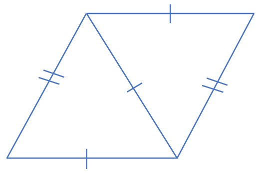
GEOMETRY REVIEW SESSION 4

For each of the following (1-4) determine if there is enough information to prove the two triangles congruent. Answer yes or no. If yes, state the theorem or postulate which would prove them congruent.

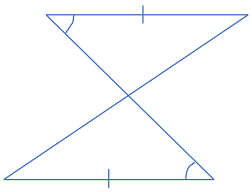
1.



2.



3.



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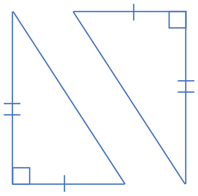
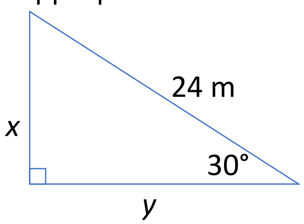
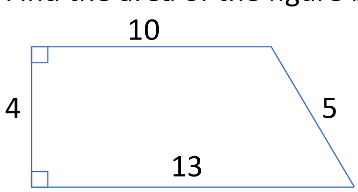


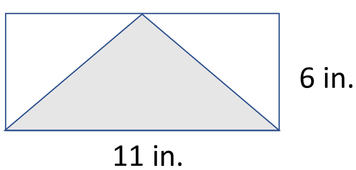
5. Without a calculator, find *x* and *y*. Leave answer in radical form if appropriate.



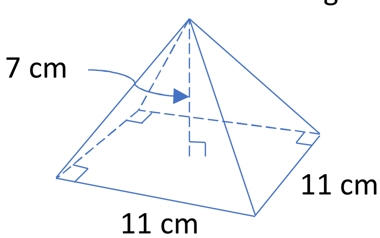
6. Find the area of the figure below.



7. Find the area of the shaded region.



8. Find the volume of the figure below.



9. Find the geometric mean between 12 and 20. Leave in radical form.

10. Find the radius of a circle with a circumference of 81.5 cm. Round your answer to the nearest tenth of a centimeter.

1.
2.
3.
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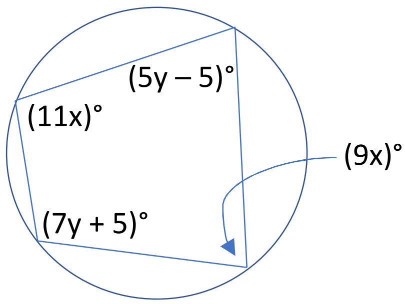
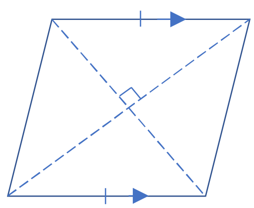
GEOMETRY REVIEW SESSION 5

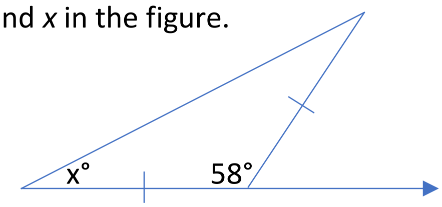
1. In the figure below, find *x and y*.



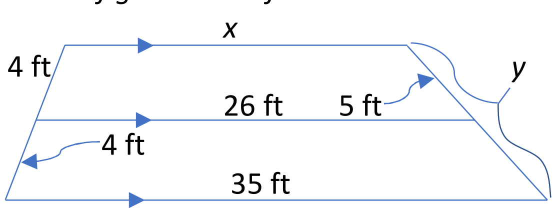
2. Name the figure below. Be as specific as possible.



3. Find *x* in the figure.



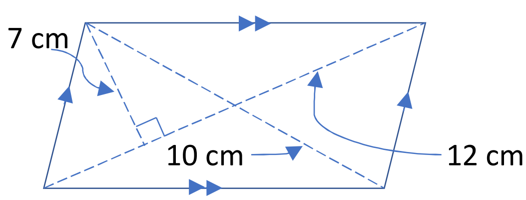
Use the figure below for 4 and 5.



4. Find *x* in the figure above.

5. Find *y* in the figure above.

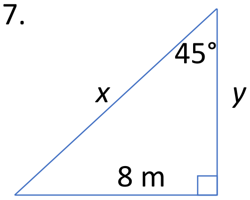
6. Find the area of the figure below.



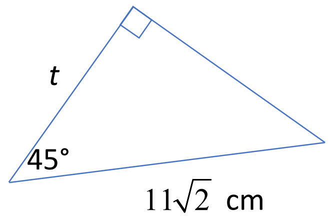
GEOMETRY REVIEW SESSION 5

In each of the following (7-10), find the missing variable(s). *Do not use a calculator in this section.*
Leave answer in radical form when appropriate.

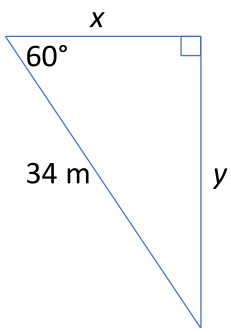
7.



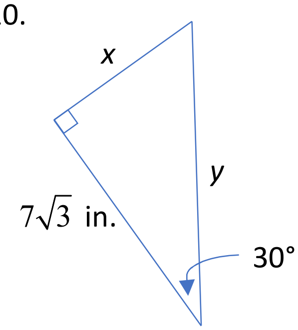
8.



9.



10.



7.
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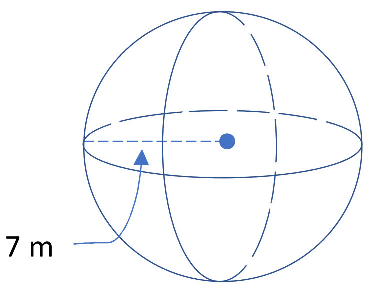
GEOMETRY REVIEW SESSION 6

1. Calculate the Surface Area and the Volume of the sphere below.



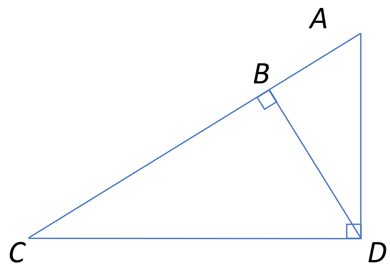
2. A rectangular building has base dimensions of 103 ft x 118 ft. The height of the building is 221 ft.
a) Find the volume of the building.

b) Find the surface area of the building. Do not count the bottom of the building in the surface area. In other words, find the outside surface area which is exposed to air. *(Note: Assume the entire building is above the ground.)*

1.
2.a. 2.b.

GEOMETRY REVIEW SESSION 6 (CONTINUED)

Use the figure below for 3-6. You may leave answers in radical form.



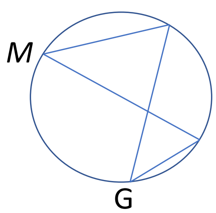
3. If AB = 7, and BC = 22, find BD.

4. If AB = 6 and BC = 18, find AD.

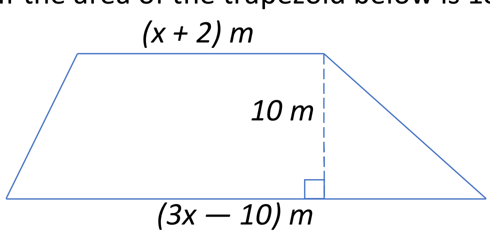
5. If AB = 4 and BC = 16, find the area of $\triangle ACD$.

6. If BD = 6 and BC = 12, find $\dfrac{Area\ of\ \triangle ABD}{Area\ of\ \triangle DBC}$.

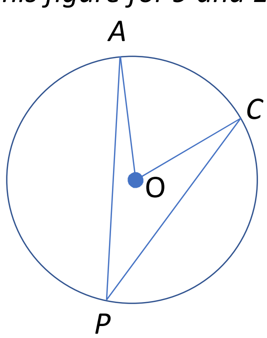
7. In the circle below, if $m\angle M = 52°$, find $m\angle G$.



8. If the area of the trapezoid below is 180 m^2, find x.



In the circle below, $m\overset{\frown}{AC} = 62°$.
Use this figure for 9 and 10.



9. Find $m\angle O$.

10. Find $m\angle P$.

3.
4.
5.
6.
7.
8.
9.
10.

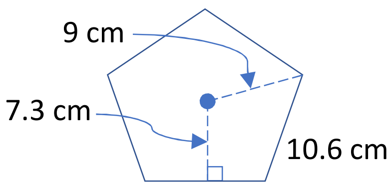
GEOMETRY REVIEW SESSION 7

A cylindrical metal drum has a diameter of 6.4 dm and a height of 9.5 dm. Use this information for 1 and 2. [A dm (decimeter) is a tenth of a meter.]

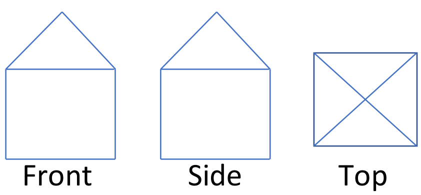
1. If the entire outside of the drum is to be painted, how many square <u>meters</u> must be painted? Round to the nearest hundredth of a m^2.

2. If the drum is to be filled with a liquid cleaner, how many liters of cleaner will it hold? (One liter is a cubic decimeter.) Round to the nearest whole number.

3. Find the area of the regular pentagon below.

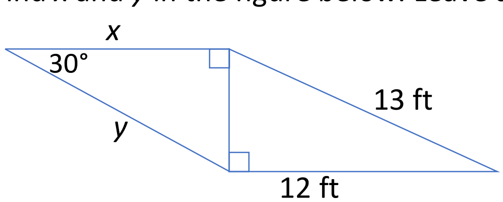


4. Draw a three-dimensional drawing of the figure, given the front, side and top views.



5. In the figure above, of what two common solids is the figure composed?

6. Find *x* and *y* in the figure below. Leave answer in radical form.



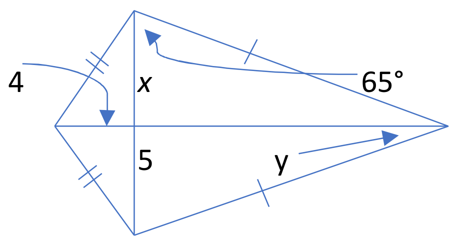
7. Answer each with *always, sometimes,* or *never.*
   a. A rectangle is a trapezoid.
   b. A rhombus is a parallelogram.
   c. A kite is a quadrilateral.
   d. A rectangle is a rhombus.

8. Which is/are not necessarily true of a rectangle?
   a. Diagonals are congruent.
   b. Diagonals are perpendicular.
   c. Opposite sides are parallel.
   d. Opposite angles are congruent.

1.
2.
3.
4.
5.
6.
7.a. 7.b. 7.c. 7.d.
8.

GEOMETRY REVIEW SESSION 7 (CONTINUED)

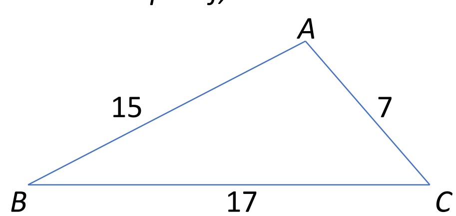
9. In the kite below, find x and y.



10. Complete the theorem: *If one pair of opposite sides of a quadrilateral are both _______ and _______, then it is a parallelogram.*

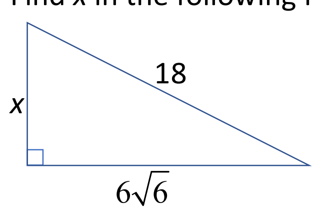
GEOMETRY REVIEW SESSION 8

Given the triangle below with side lengths shown, complete the steps of the indirect proof, 1-3.

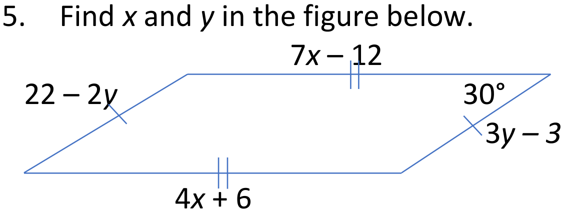


Prove that $\angle A$ *is not a right angle.*

1. Assume: _________________.
2. Then $15^2 + 7^2 = 17^2$, by the _________________ Theorem.
3. But this is not true. Therefore, our assumption is false, and ________________________.
4. Find x in the following figure. Write answer in simplified radical form.



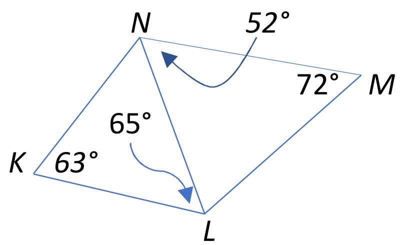
5. Find x and y in the figure below.



6. Use the information from #5 above and find the area of the figure. *Hint: Use a 30°-60°-90° triangle.*

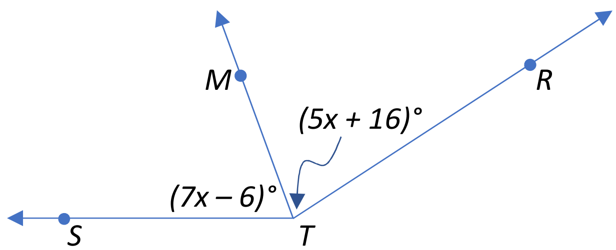
GEOMETRY REVIEW SESSION 8 (CONTINUED)

7. Name the longest segment in the figure below. Explain.
Caution: Figure is not drawn to scale!



7.

8. In the figure below, $\overrightarrow{TM}$ bisects $\angle STR$. Find *x* <u>and</u> $m\angle STR$.

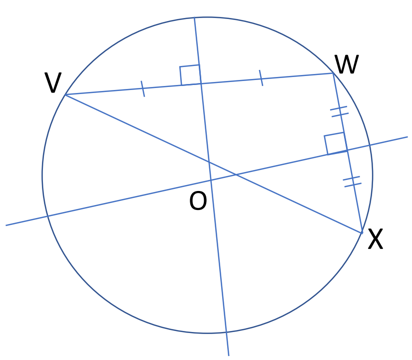


8.

9. Find the midpoint of the segment with endpoints $(-4, 2)$ and $(6, -10)$.

9.

10. Choose the correct description of point O.



a. centroid of $\triangle VWX$
b. incenter of $\triangle VWX$
c. orthocenter of $\triangle VWX$
d. circumcenter of $\triangle VWX$

10.

GEOMETRY REVIEW SESSION 9 (ALGEBRA RECAP)

1. Determine the slope of a line passing through $(3,-1)$ and $(5,7)$.

2. Write the slope-intercept form of the equation of the line which passes through $(4,7)$ and is perpendicular to $y=-\frac{1}{2}x+4$.

3. Solve the linear system.

$$2x+y=9$$
$$4x-3y=-7$$

4. Write the equations of **a)** *horizontal* and **b)** *vertical* lines passing through $(-6,4)$.

For 5 and 6, simplify the expression and write without negative exponents.

5. $\left(x^{-3}y^{2}\right)^{2}$

6. $\dfrac{4ac^{-4}}{5a^{-1}b^{-3}}$

7. Evaluate the expression. Write the answer in **a)** scientific notation and **b)** standard numerical form.

$$\frac{8\times10^{-5}}{2.5\times10^{3}}$$

For 8-9, factor each expression completely.

8. $4x^{2}-25$

9. $2x^{2}-5x-12$

10. *Calculator recommended.* Solve the linear system.

$$y=-2x-\frac{1}{2}$$
$$y=\frac{6}{5}x-\frac{9}{2}$$

1.
2.
3.
4.a. 4.b.
5.
6.
7.a. 7.b.
8.
9.
10.

GEOMETRY REVIEW SESSION 10 (ALGEBRA RECAP)

For 1-2, simplify the expression, leaving no radical in the denominator.

1. $\sqrt{\dfrac{48}{15}}$

2. $\dfrac{2}{15}\sqrt{75}\sqrt{3}$

3. Solve the equation using the quadratic formula.

   $2x^2 - 3x - 7 = 0$

4. Find the product.

   $(3x-5)^2$

5. Martha and Tony invest $22,000 in equipment and workspace to make outdoor planters. The materials and labor to make each planter is $17.50.

   a) Write a linear model for the total cost of production. Let *C* be the total cost and *x* the number of planters produced.

   b) What is the total cost to make 45,000 planters?

6. Solve.

   $\dfrac{x-3}{10} = \dfrac{9}{x-2}$

7. Solve and graph the inequality.

   $|8-3x| \geq 10$

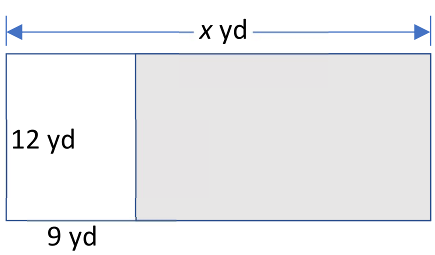
8. Solve.

   $0 = (3-2x)(5x+8)$

9. Simplify.

   $(2x-5)-(x^2-6x+7)$

10. Write a variable expression for the area of the shaded portion of the figure below.



1.
2.
3.
4.
5.a. 5.b.
6.
7. 
8.
9.
10.

END OF REVIEW